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Information and investment under uncertainty

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HIGHLIGHTS

- Investment decisions depend both on return and non-return information.
- Availability of non-return information affects the flow-return relationship.
- Non-return information affects investors' participation decisions.
- Fund managers release all non-return information when releasing is not costly.
- Fund managers release only good non-return information when releasing is costly.

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ABSTRACT

In a model of mutual fund investor learning about managerial ability, we investigate how investment decisions depend on both past returns and non-return information. We show that non-return information affects investors' reliance on past returns as well as their decision to participate. Our results have important consequences for the relationship between flows of money and past fund performance.

1. Introduction

Rational investors rely on all relevant information when making investment decisions under uncertainty. Relevant information includes past returns on the same or similar investments, but also non-return information such as that contained in analysts' reports, advertising material for investors, and news articles. In this paper we ask: How does the availability of non-return information affect investors' decisions and, in particular, investors' reliance on past returns?

The open-end structure of mutual funds provides an ideal setting to study investment decisions. Mutual fund shares are issued and redeemed at investors' request and their price is set equal to the value of the fund's net assets per share, so it does not adjust to changes in demand and supply. Therefore, mutual fund flows provide a rare insight into investors' motives for trading. Many studies have investigated the determinants of mutual fund flows (see Christoffersen et al., 2014, for a survey). Two stylized facts have emerged from this literature. First, investors' demand for mutual funds is increasing in past performance, consistently with rational investors learning from past returns about future fund performance (Huang et al., 2012). Second, the relationship between mutual fund flows and past performance appears to be convex (Sirri and Tufano, 1998).

We propose a model of mutual fund flows in which Bayesian investors learn from both past returns and publicly observable non-return information about managerial ability. We build on the model of Huang et al. (2007) and distinguish between existing and new investors of a given fund. Existing investors have more

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precise priors about managerial ability than new investors. While existing investors update their priors upon observing past returns, new investors must first decide whether they wish to pay a participation cost to improve their priors. We depart from Huang et al. (2007) in that both existing and new investors observe a public signal about managerial ability whose realization for a given fund is independent from past returns.

As in Huang et al. (2007), in our model mutual fund flows are increasing and convex in past returns. Conditional on past returns, flows increase (decrease) with a favorable (unfavorable) realization of the public non-return signal about managerial ability. Investors' reliance on the non-return signal increases with its precision *relative* to that of past returns. Finally, we show that non-return information not only affects the level of flows conditional on past returns, but it also changes the shape of the flow-performance relationship in fundamental ways.

Like other models of mutual fund investor learning, ours assumes that information on past returns is readily available to investors. However, non-return information (such as fees, the manager's track record, or fund ratings produced by third parties) is often less ubiquitous and many investors learn it only through mutual fund advertising. To account for this distinction between return and non-return information, we study a variant of the model in which only the manager observes the non-return signal and can release it to the public at a cost. Therefore, the manager can censor, but not manipulate, the non-return signal. We show that when the cost of releasing the signal is sufficiently high, in equilibrium the manager chooses to release it only if it is favorable enough.

A number of empirical studies have looked at mutual fund investors' reaction to non-return information such as media mentions or advertising (Sirri and Tufano, 1998; Jain and Wu, 2000). Our model provides a novel interpretation of previous empirical results. For instance, Sirri and Tufano (1998) investigate whether advertising and media mentions of mutual funds increase both flows to mutual funds and the sensitivity of flows to past returns, consistently with the idea that more information reduces search costs. They find supporting evidence for the former hypothesis but not the latter. Based on our analysis, one possible explanation for their results is that when non-return information about mutual funds becomes available, past returns become relatively less informative about future returns, which weakens the flow-performance relationship. More generally, our paper advises that future tests of hypotheses related to the flow-performance relationship take into account the availability and content of non-return information, as failure to do so could bias inference if non-return information changes in the time series or cross-section. Finally, to the extent that the flow-performance relationship generates incentives for managers (Chevalier and Ellison, 1997), our model predicts that such incentives will vary with the availability of public non-return information.

2. The model

We consider an economy where investors allocate their wealth between several actively managed funds $i = 1, \ldots, I$ and a risk-free asset (whose return is normalized to zero). There are three dates: $t = 0, 1, 2$. The mutual fund i produces a risky return

$$r_{it} = \alpha_i + \varepsilon_{it}, \quad t = 1, 2 \text{ and } i = 1, \ldots, I,$$

where α_i is the unobservable ability of fund i's manager, $\alpha_i \sim N\left(\alpha_{i0}, \sigma_0^2\right)$, and $\varepsilon_{it} \sim N\left(0, \sigma_\varepsilon^2\right)$ represents the idiosyncratic noise in the return of fund i, and is independently and identically distributed both through time and across funds. As in Huang et al. (2007), there are two types of investors for each fund: existing investors and new investors. There is a population normalized to 1 of existing investors and λ_i of new investors.

Both existing and new investors observe past returns, and infer managerial ability from them. While existing investors know the manager's expected ability level α_{i0}, new investors only know its distribution: $\alpha_{i0} \sim N\left(\mu_0, \sigma_0^2\right)$. Note that the mean, μ_0, and the variance of the expected ability of the fund's manager, σ_0^2, are known to all investors. To learn the expected ability of fund i's manager, α_{i0}, a new investor k must pay a participation cost $c_{ki} \geq 0$. We assume that $c_{ki} = \delta_k \overline{c}_i$, where $\delta_k \sim Unif[0, 1]$ captures the level of new investors' sophistication and $\overline{c}_i$ reflects the variation across funds in the difficulty for investors to narrow down an uninformative prior for α_{i0}.[1]

We depart from Huang et al. (2007) by assuming that both existing investors and new investors observe a public non-return signal about managerial ability, $S_i = \alpha_i + e_i$, for any $i = 1, \ldots, I$, where $e_i \sim N\left(0, \sigma_e^2\right)$ and e_i and ε_{it} are independent across funds and time.

All investors have the same initial wealth W_1 and CARA preferences over their terminal wealth W_2, with risk aversion coefficient γ. Since investors observe both the fund's past return r_{i1} and the non-return signal S_i, their expected utility is

$$E\left[U\left(W_2\right) \middle| r_{i1}, S_i\right] = E\left[-e^{-\gamma W_2} \middle| r_{i1}, S_i\right], \quad \text{where } W_2 = W_1 + \sum_{i=1}^{I} X_{i1} r_{i2}.$$

At date $t = 1$ the funds' returns, r_{i1}, $i = 1, \ldots, I$ are realized and observed by investors together with the non-return signals, S_i, $i = 1, \ldots, I$. After observing returns and non-return signals, a new investor decides whether to pay the participation cost c_{ki} to learn α_{i0}. As in Huang et al. (2007) we assume that if an investor k does not pay the participation cost c_{ki} corresponding to fund i, she makes no investment in that fund. We assume that existing investors have an initial stake in fund i at $t = 0$, which we denote by X_{i0}. X_{i1}^e and X_{i1}^n denote the optimal holding in fund i at $t = 1$ of existing and new investors, respectively.

We determine first the optimal holdings of investors in fund i conditional on participating in that fund. Note that investors cannot short-sell the fund, so holdings must be positive.

Lemma 1. *The optimal holdings of existing and new investors in fund i are*

$$X_{i1}^e = X_{i1}^n = X_{i1}\left(r_{i1}, S_i\right) = \begin{cases} \dfrac{Z_1}{\gamma V}\alpha_{i0} + \dfrac{Z_2}{\gamma V} r_{i1} + \dfrac{Z_3}{\gamma V} S_i, \\ \quad \text{if } \left(\sigma_e^2 r_{i1} + \sigma_\varepsilon^2 S_i\right) \geq -\dfrac{\sigma_\varepsilon^2 \sigma_e^2 \alpha_{i0}}{\sigma_0^2} \\ 0, \quad \text{if } \left(\sigma_e^2 r_{i1} + \sigma_\varepsilon^2 S_i\right) < -\dfrac{\sigma_\varepsilon^2 \sigma_e^2 \alpha_{i0}}{\sigma_0^2}, \end{cases}$$

where $V \equiv \sigma_\varepsilon^2 \left(2\sigma_0^2\sigma_e^2 + \sigma_\varepsilon^2\left(\sigma_0^2 + \sigma_e^2\right)\right)$, $Z_1 \equiv \sigma_\varepsilon^2\sigma_e^2$, $Z_2 \equiv \sigma_0^2\sigma_e^2$ *and* $Z_3 \equiv \sigma_0^2\sigma_\varepsilon^2$.

Investors decide how much to invest in fund i based on the two signals about managerial ability: the past return and the non-return signal. The relative weight they attach to each signal

[1] In this paper we extend the model of Huang et al. (2007), in which fund performance net of transaction costs is not affected by investors' decisions. Alternatively, we could solve for an equilibrium à la Berk and Green (2004) in which fund inflows negatively affect fund performance through diseconomies of scale in portfolio management, so all funds offer a zero expected net return in equilibrium. The main insights of the model regarding the consequences of the non-return signal for flows would not change. However, in such an equilibrium new investors would never want to participate.

depends on their relative precision $\rho \equiv \sigma_e^2/\sigma_\varepsilon^2$. If $\rho < 1$ then S_i is more precise than r_{i1}, so investors put a higher weight on it, and vice versa.

To derive the optimal participation decision of a new investor k in fund i, we compare the certainty equivalent of wealth gain from investing in that fund with the participation cost c_{ki}.

Lemma 2. *The certainty equivalent of wealth gain for investing in fund i for an investor that observes signals r_{i1} and S_i is*

$$g\left(B\left(r_{i1}, S_i\right)\right) = -\frac{1}{\gamma}\ln\left(\frac{1-erf\ (B)}{2} + \frac{1+erf\ \left(\frac{B}{A}\right)}{2A} \times \exp\left(-\left(1-\frac{1}{A^2}\right)B^2\right)\right),$$

where $erf\ (x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$ *is the error function and*

$$A \equiv \sqrt{1+\frac{\sigma_\mu^2 Z_1^2}{FV}},$$

$$B \equiv \frac{Z_1\mu_0 + Z_2 r_{i1} + Z_3 S_i}{\sqrt{2}\sigma_\mu Z_1},$$

$$F \equiv \sigma_0^2\sigma_e^2 + \left(\sigma_0^2+\sigma_e^2\right)\sigma_\varepsilon^2.$$

A new investor k participates in fund i whenever $g\left(B\left(r_{i1}, S_i\right)\right) \geq c_{ki}$. Let us define B^* such that $g\left(B^*\right) = c_{ki}$. Since $g\left(B\right)$ is increasing in B, for any $\left(r_{i1}, S_i\right)$ for which $B\left(r_{i1}, S_i\right) > B^*$, the investor will participate in fund i. Notice that r_{i1} and S_i are substitutes. New investors decide to participate whenever the combination of the two signals leads them to infer that the expected performance is high enough to make the certainty equivalent of wealth gain higher than the participation cost.

We define the net flow as the increase in the fund's assets net of the fund's profit during the period, and as a fraction of initial assets. We calculate it by aggregating the holdings of existing investors and participating new investors.

Proposition 1. *The net flow in fund i equals to*

$$Flows\left(r_{i1}, S_i\right) = \frac{X_{i1}^e\left(r_{i1}, S_i\right) - X_{i0}\left(1+r_{i1}\right)}{X_{i0}} + \lambda_i \min\left\{1, \frac{g\left(r_{i1}, S_i\right)}{\bar{c}_i}\right\}\frac{X_{i1}^n\left(r_{i1}, S_i\right)}{X_{i0}}.$$

The net flow into fund i is increasing and convex in past performance r_{i1}.[2]

In Fig. 1, we plot flows as a function of past returns for different parameter sets. In the presence of participation costs, $\bar{c} > 0$, we obtain a similar flow-performance relationship to Huang et al. (2007) in the sense that flows are increasing in past returns (conditional on investors willing to hold positive shares of the fund) and convex in the presence of participation costs. Convexity arises because higher returns both induce more new investors to participate and increase optimal holdings of participating investors.

[2] Note that, exactly as Huang et al. (2007), we employ the standard definition of net inflows used in the literature, i.e., dollar inflows divided by assets under management at the beginning of the period. When expected performance is too low, the optimal investment in the fund is zero, and investors withdraw all their money from the fund. In this case, net flows equal $-\left(1+r_{i1}\right)$, so they are decreasing in returns: Holding initial assets constant, assets to be withdrawn from the fund are larger the higher the fund's past return.

However, flows in our model also depend on non-return information: Favorable non-return information increases fund flows, conditional on past returns. The sensitivity of flows to the non-return signal increases with its precision relative to that of the return signal.

The presence of a non-return signal about managerial ability not only changes the level of flows but also the shape of the flow-return relationship in three different ways. First, non-return information makes investors put less weight on past performance when making investment decisions, so the presence of a non-return signal tends to decrease the sensitivity of flows to past returns.

Second, a favorable non-return signal about managerial ability induces investors to invest in the fund for a lower level of past returns. Consequently, funds with poor past performance that would otherwise be shun by investors, may be in positive demand in the presence of favorable non-return information.

Third, in the presence of participation costs, a favorable non-return signal increases the expected benefit of participating for new investors, so they pay the participation cost for a lower level of past returns. Therefore, the sensitivity of flows to past returns increases with favorable non-return news about the fund. The effect is larger, the larger the relative precision of the non-return signal.

3. Strategic information transmission

Like other models of mutual fund investor learning, ours assumes that information on past returns is readily available to investors at no cost. Indeed, returns (net of expenses) are reported not just in fund prospectuses, but are also published daily online and in print, and are the basis of fund rankings. The assumption of observability, however, is less realistic when we think about other relevant information about future fund performance. One such example are fund fees. Although investors can actively look for information on fees, doing so requires effort, time, and some knowledge on their part. Consequently, many investors will learn about fees only when the management company decides to advertise them. Other examples of information that the management company may choose to advertise to investors include the track record and other information about the fund manager, measures of relative performance, and fund ratings produced by third parties.

To model the distinct nature of return and non-return signals, we allow the fund manager to have discretion over the release of the non-return signal S_i. Our set-up is similar to that of Mullainathan et al. (2008), who develop a model of strategic transmission of information and study mutual fund fee advertising as an extension of the model.[3] We assume that the non-return signal is always credible and it is hard information, i.e., it cannot be manipulated, but it can be censored. Thus, the manager engages in a persuasion game and releases the signal S_i only if the realization is such that the flows the fund receives when investors observe the signal S_i net of the cost of releasing the signal normalized by the fund fee ($K \geq 0$) are higher than the flows when the signal is not released.

Let us denote by $Flows(r_{i1}, S_i)$ net inflows to fund i if signal S_i is released, by $Flows(r_{i1}, \varnothing)$ net inflows to fund i if no non-return signal is released, and by $Flows(r_{i1})$ net inflows to fund i when only the return signal r_{i1} can be released, i.e., the case studied by Huang et al. (2007). Note that $Flows(r_{i1}, S_i)$ are the inflows to fund i in the case when there is no strategic decision regarding the release of the non-return signal S_i i.e., exactly the model described in Section 2.

[3] This extension is included in the working paper version of Mullainathan et al. (2008).

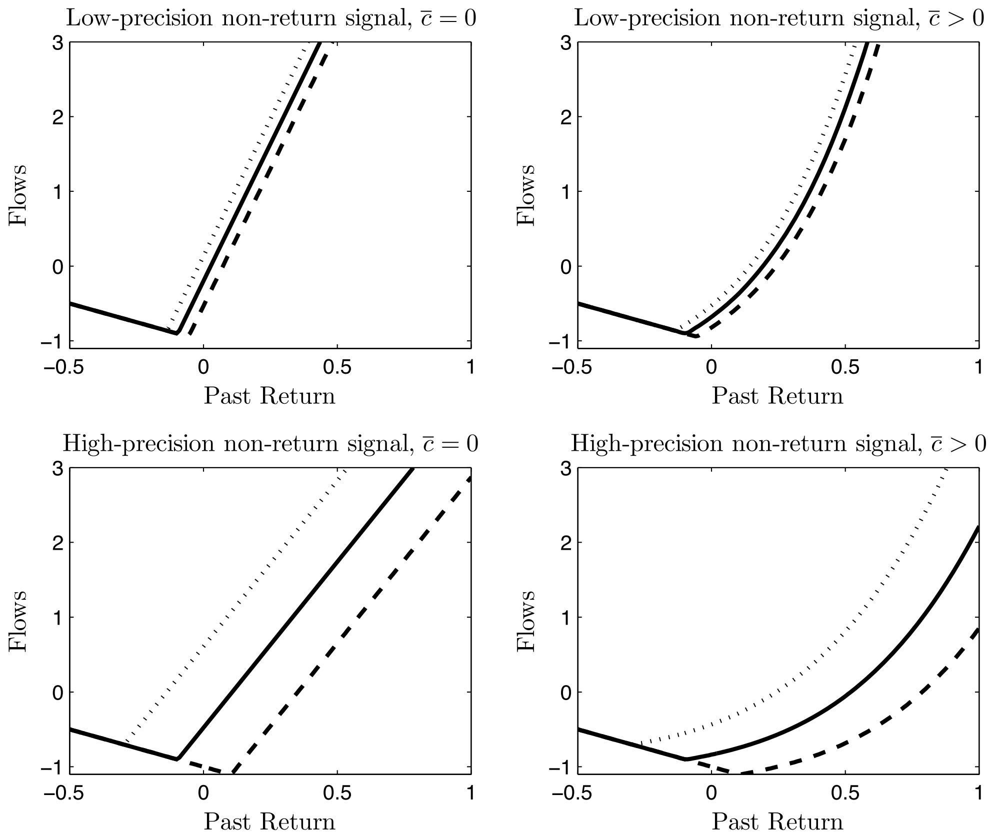

Fig. 1. Comparison of fund flows for different precisions and realizations of the non-return signal. The dotted line corresponds to a high realization of the non-return signal, the solid line to a neutral realization and the dashed line to a low realization. Parameter values: $\mu_0 = 0.03, \alpha_0 = 0.03, \sigma_0^2 = 0.03, \sigma_\varepsilon^2 = 0.16, \gamma = 1, \lambda = 0.5, X_0 = 0.5, \bar{c} = 0.1$ (when $\bar{c} > 0$), $\rho = 0.25$ for the high precision non-return signal and $\rho = 1.25$ for the low precision non-return signal.

We consider two cases depending on whether investors understand or not the strategic decision of the manager to release the non-return signal when updating their beliefs.

Face-value investors

Investors do not recognize the endogeneity of the decision to release the non-return signal and take messages at face value. If no signal realization is released, investors just assume there is no non-return information. Therefore, the fund manager has incentives to release S_i whenever:

$$Flows(r_{i1}, S_i) - K \geq Flows(r_{i1}).$$

We define $\bar{S}_{FV}$ the cut-off point such that

$$Flows(r_{i1}, \bar{S}_{FV}) - K = Flows(r_{i1}). \qquad (1)$$

Consider an equilibrium in which the optimal decision of the fund manager is as follows:

$$\begin{cases} \text{Release the signal } S_i & \text{if} \quad S_i \geq \bar{S}_{FV}, \\ \text{Do not release} & \text{otherwise.} \end{cases}$$

Note that since $Flows(r_{i1}, S_i)$ is increasing in S_i for all $K \geq 0$ and $Flows(r_{i1})$ does not depend on S_i, for any r_{i1} there exists $\bar{S}_{FV}$ that satisfies (1). Therefore the equilibrium described above exists.

Rational investors

Investors recognize the strategic nature of the decision to release the non-return signal S_i and assume a poor realization of the non-return signal if no value of the signal is released. When the cost of releasing the signal is $K = 0$, in equilibrium the fund manager always releases the signal (this result is similar to the unraveling result of Grossman, 1981 and Milgrom, 1981).

To prevent the unraveling equilibrium, we need to impose some cost $K > 0$ to the fund manager for disclosing the non-return signal (see Verrecchia, 1983). In this case, the manager may find it optimal not to disclose the signal and save K when the signal realization is below some threshold even if that means that investors will infer that the signal realization is below the threshold. Consider an equilibrium in which the optimal decision of the fund manager is the following

$$\begin{cases} \text{Release the signal } S_i & \text{if} \quad S_i \geq \bar{S}_R, \\ \text{Do not release} & \text{otherwise} \end{cases} \qquad (2)$$

where $\bar{S}_R$ is such that

$$Flows(r_{i1}, \bar{S}_R) - K = Flows(r_{i1}, \varnothing). \qquad (3)$$

As explained in the Appendix, for K sufficiently high and r_{i1} finite, there exists $\bar{S}_R$ that satisfies (3), and therefore there exists an equilibrium as that described in (2).

Note that when the realization of the non-return signal is released, all the results for the case with no strategic release of information by the fund manager hold. Therefore, the depiction of funds flows with strategic information transmission and rational investors is similar to that in Fig. 1. For any non-return signal $S_i \geq \bar{S}_R$, the fund flows are $Flows(r_{i1}, S_i)$, while for any signal $S_i < \bar{S}_R$ the fund flows are equal to $Flows(r_{i1}, \bar{S}_R) - K$. Consequently, there is a discontinuity in flows at $S_i = \bar{S}_R$: Flows will jump when relevant fund information is advertised.

Our model provides an explanation for why fund managers may decide to drop information when such information is bad. Mullainathan et al. (2008) also find that mutual fund managers may strategically choose to release only good signals about future fund performance (such as low management fees). However, in their model such result arises when investors exhibit coarse thinking (they use associative strategies when evaluating different propositions), while our result is true for rational investors.

4. Conclusions

We investigate how mutual fund investors' decisions change when investors can learn about future performance from non-return information. We show that the availability of non-return information not only impacts investor choices given past returns, but it also reduces investors' reliance on past returns as a source of information. Moreover, non-return information changes the return threshold that induces existing investors to purchase the fund and new investors to pay the participation cost. As a consequence, the much-studied shape of the flow-performance relationship changes in fundamental ways.

Our results warn researchers against ignoring the impact of non-return information on investor decisions when studying the determinants of the flow-performance relationship, as the variables of interest could be correlated with the availability of non-return information.

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Appendix

Proof of Lemma 1. The maximization problem of both existing and new investors, conditional on investors participating is

$$\max_{X_{i1}\geq 0, i=1,\ldots,I} E\left[U\left(W_2\right)\middle| r_{i1}, S_i\right] = E\left[-e^{-\gamma W_2}\middle| r_{i1}, S_i\right]$$

$$W_2 = W_1 + \sum_{i=1}^{I} X_{i1} r_{i2}$$

$$\Leftrightarrow \max_{X_{i1}\geq 0, i=1,\ldots,I}\left(W_1 + \sum_{i=1}^{I} X_{i1} E\left(r_{i2}\middle| r_{i1}, S_i\right) - \frac{1}{2}\gamma\sum_{i=1}^{I} X_{i1}^2 Var\left(r_{i2}\middle| r_{i1}, S_i\right)\right).$$

Due to the CARA properties and the fact that returns and signals are independent across funds, the problem can be solved independently for each fund i. The demand for fund i is

$$X_{i1} = \frac{E\left(r_{i2}\middle| r_{i1}, S_i\right)}{\gamma Var\left(r_{i2}\middle| r_{i1}, S_i\right)}, \quad \forall i = 1, \ldots, I. \tag{4}$$

Applying the projection theorem for normally distributed random variables,

$$X_{i1} = \frac{E\left(r_{i2}\middle| r_{i1}, S_i\right)}{\gamma Var\left(r_{i2}\middle| r_{i1}, S_i\right)} = \frac{Z_1\alpha_{i0} + Z_2 r_{i1} + Z_3 S_i}{\gamma V},$$

where $V = \sigma_\varepsilon^2\left(2\sigma_0^2\sigma_e^2 + \sigma_\varepsilon^2\left(\sigma_0^2 + \sigma_e^2\right)\right)$, $Z_1 = \sigma_\varepsilon^2\sigma_e^2, Z_2 = \sigma_0^2\sigma_e^2$ and $Z_3 = \sigma_0^2\sigma_\varepsilon^2$.

Due to the short-sale constraint, the demand for fund i is $X_{i1} \geq 0$, i.e., $Z_1\alpha_{i0} + Z_2 r_{i1} + Z_3 S_i \geq 0$ or equivalently,

$$Z_2 r_{i1} + Z_3 S_i \geq -Z_1\alpha_{i0}.$$

If this condition does not hold, demand equals 0. ■

Proof of Lemma 2. We use the solution in (4) to compute the final wealth

$$W_2 = W_1 + \sum_{i=1}^{I} X_{i1} r_{i2} = W_1 + \sum_{i=1}^{I}\frac{E\left(r_{i2}\middle| r_{i1}, S_i\right)}{\gamma Var\left(r_{i2}\middle| r_{i1}, S_i\right)} r_{i2}.$$

The certainty equivalent for investing in fund i is therefore,

$$E\left[U\left(W_2\right)|r_{i1}, S_i\right] = E\left[-e^{-\gamma W_2}|r_{i1}, S_i\right] = -\exp\left(-\gamma W_1\right)\prod_{i=1}^{I}\exp\left(-\frac{1}{2}\times\sum_{i=1}^{I}\frac{\left(E\left(r_{i2}\middle| r_{i1}, S_i\right)\right)^2}{Var\left(r_{i2}\middle| r_{i1}, S_i\right)}\right).$$

The certainty equivalent of wealth gain for investing in fund i is

$$\exp(-\gamma\left(g\left(r_{i1}, S_i\right) - c_{ki}\right)) = E\left[e^{-\gamma\left(X_{i1}r_{i2} - c_{ki}\right)}|r_{i1}, S_i\right], \tag{5}$$

and an investor k will invest in the fund i if and only if $g\left(r_{i1}, S_i\right) - c_{ki} \geq 0$.

$$\exp\left(-\gamma\left(g\left(r_{i1}, S_i\right) - c_{ki}\right)\right) = E\left[e^{-\gamma\left(X_{i1}r_{i2} - c_{ki}\right)}|r_{i1}, S_i\right] = e^{\gamma c_{ki}}\left(I_1 + I_2\right),$$

where

$$I_1 \equiv \int_{-\infty}^{\bar{a}} f\left(\alpha_{i_0}\right) d\alpha_{i0} = \frac{1}{2} - \frac{1}{2} erf\left(B\right),$$

and

$$I_2 \equiv \int_{\bar{a}}^{\infty} e^{-\frac{1}{2}\frac{\left(E\left(r_{i2}|r_{i1}, S_i\right)\right)^2}{Var\left(r_{i2}|r_{i1}, S_i\right)}} f\left(\alpha_{i_0}\right) d\alpha_{i0} = \frac{1}{2A}\exp\left(-\left(1 - \frac{1}{A^2}\right)B^2\right)\left(1 + erf\left(\frac{B}{A}\right)\right),$$

where we define

$$\bar{a} \equiv -\frac{Z_2 r_{i1} + Z_3 S_i}{Z_1},$$

$$A \equiv \sqrt{1 + \frac{\sigma_\mu^2 Z_1^2}{FV}},$$

$$B \equiv \frac{Z_1\mu_0 + Z_2 r_{i1} + Z_3 S_i}{\sqrt{2} Z_1\sigma_\mu}.$$

From (5):

$$\exp\left(-\gamma\left(g\left(r_{i1}, S_i\right) - c_{ki}\right)\right) = e^{\gamma c_{ki}}\left(I_1 + I_2\right), \quad \text{where}$$

$$g\left(r_{i1}, S_i\right) = -\frac{1}{\gamma}\ln\left(I_1 + I_2\right) = -\frac{1}{\gamma}\ln\left(\frac{1 - erf\left(B\right)}{2} + \frac{1 + erf\left(\frac{B}{A}\right)}{2A}\exp\left(-\left(1 - \frac{1}{A^2}\right)B^2\right)\right). \quad ■$$

Strategic information transmission. The net flow in fund i equals to

$$Flows\left(r_{i1}, S_i\right) = \frac{X_{i1}^e\left(r_{i1}, S_i\right) - X_{i0}\left(1 + r_{i1}\right)}{X_{i0}} + \lambda_i\min\left\{1, \frac{g\left(r_{i1}, S_i\right)}{\bar{c}_i}\right\}\frac{X_{i1}^n\left(r_{i1}, S_i\right)}{X_{i0}}.$$

The net flow into fund i is increasing and convex in past performance r_{i1}.

$$Flows\left(r_{i1}\right) = \frac{\hat{X}^{e}_{i1}\left(r_{i1}\right) - X_{i0}\left(1 + r_{i1}\right)}{X_{i0}} + \lambda_i \min\left\{1, \frac{g\left(r_{i1}\right)}{\bar{c}_i}\right\} \frac{\hat{X}^{n}_{i1}\left(r_{i1}\right)}{X_{i0}},$$

where $\hat{X}^{e}_{i1}\left(r_{i1}\right)$, $\hat{X}^{n}_{i1}\left(r_{i1}\right)$, and $g\left(r_{i1}\right)$ are the optimal investment by existing and new investors, and the certainty equivalent, respectively, in the model of Huang et al. (2007).

As a result, $\bar{S}_{FV}$ satisfies

$$X^{e}_{i1}\left(r_{i1}, \bar{S}_{FV}\right) + \lambda_i \min\left\{1, \frac{g\left(r_{i1}, \bar{S}_{FV}\right)}{\bar{c}_i}\right\} X^{n}_{i1}\left(r_{i1}, \bar{S}_{FV}\right) - KX_{i0} = \hat{X}^{e}_{i1}\left(r_{i1}\right) + \lambda_i \min\left\{1, \frac{g\left(r_{i1}\right)}{\bar{c}_i}\right\} \hat{X}^{n}_{i1}\left(r_{i1}\right).$$

Similarly, $\bar{S}_R$ satisfies

$$X^{e}_{i1}\left(r_{i1}, \bar{S}_{R}\right) + \lambda_i \min\left\{1, \frac{g\left(r_{i1}, \bar{S}_{R}\right)}{\bar{c}_i}\right\} X^{n}_{i1}\left(r_{i1}, \bar{S}_{R}\right) - KX_{i0} = X^{Se}_{i1}\left(r_{i1}, \bar{S}_{R}\right) + \lambda_i \min\left\{1, \frac{g^{S}\left(r_{i1}, \bar{S}_{R}\right)}{\bar{c}_i}\right\} X^{Sn}_{i1}\left(r_{i1}, \bar{S}_{R}\right),$$

where $X^{Se}_{i1}\left(r_{i1}, \bar{S}_{R}\right)$, $X^{Sn}_{i1}\left(r_{i1}, \bar{S}_{R}\right)$ are the optimal investment quantities for the existing and new investors in the case of strategic information transmission:

$$\max_{X^{S}_{i1} \geq 0, i=1,\ldots,I} E\left[U\left(W_2\right) \middle| r_{i1}, S_i < \bar{S}_R\right] = E\left[-e^{-\gamma W_2} \middle| r_{i1}, S_i < \bar{S}_R\right]$$

$$W_2 = W_1 + \sum_{i=1}^{I} X^{S}_{i1} r_{i2}.$$

Note that in this case $r_{i2} | \left(r_{i1}, S_i < \bar{S}_R\right)$ is not conditionally normally distributed and we have to use numerical methods to obtain the optimal investment quantities. Similarly, we have to use numerical methods to solve for the certainty equivalent of wealth gain for investing in fund i

$$\exp(-\gamma\left(g^{S}\left(r_{i1}, \bar{S}_R\right) - c_{ki}\right)) = E\left[e^{-\gamma\left(X^{S}_{i1} r_{i2} - c_{ki}\right)} \middle| r_{i1}, S_i < \bar{S}_R\right].$$

The conditional density of r_{i2} given r_{i1} and $S_i < \bar{S}_R$

$$f_{r_{i2}|r_{i1}, S_i < \bar{S}_R}\left(r_{i1}, r_{i2}, S_i\right) = \frac{\int_{-\infty}^{\bar{S}_R} f\left(r_{i1}, r_{i2}, S_i\right) dS_i}{\int_{-\infty}^{\infty} \int_{-\infty}^{\bar{S}_R} f\left(r_{i1}, r_{i2}, S_i\right) dr_{i1} dS_i} = \frac{\int_{-\infty}^{\bar{S}_R} \exp\left\{-\frac{1}{2}\left(W - \mu\right)' \Sigma^{-1}\left(W - \mu\right)\right\} dS_i}{\int_{-\infty}^{\infty} \int_{-\infty}^{\bar{S}_R} \exp\left\{-\frac{1}{2}\left(W - \mu\right)' \Sigma^{-1}\left(W - \mu\right)\right\} dr_1 dS_i},$$

where $W = \begin{pmatrix} r_{i2} \\ r_{i1} \\ S_i \end{pmatrix}$, $f\left(r_{i1}, r_{i2}, S_i\right)$ is the joint density probability density function, and $W \sim N\left(\mu, \Sigma\right)$.

Let us now discuss the existence of $\bar{S}_R$. We know that

$$\lim_{S_i \to -\infty} Flows(r_{i1}, S_i) = \lim_{\bar{S}_R \to -\infty} Flows(r_{i1}, \varnothing) = -\left(1 + r_{i1}\right),$$

and this is due to the participation constraint of the investors. Note also that $Flows(r_{i1}, S_i)$ is increasing and convex in S_i and $\lim_{S_i \to \infty} Flows(r_{i1}, S_i) = +\infty$. On the other hand, we have that $\lim_{\bar{S}_R \to \infty} Flows(r_{i1}, \varnothing) = Flows\left(r_{i1}\right) < \infty$. Note that this implies that for K sufficiently high, there exists a unique solution of the Eq. (3). So in this case we have an equilibrium in which there is a cut-off above which the fund manager decides to release the non-return signal S_i and below which he decides to drop it. ■

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