# The Value of the "Swap" Feature in Equity Default Swaps*

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**Abstract**

When equity default swap (EDS) contracts were first included in a rated collateralized debt obligation (CDO) deal, some critics doubted the originality of the product. In fact, EDSs are equivalent to already existing binary barrier options on equity, except the premium is not paid upfront, but over time, and conditional on the trigger event not having occurred. Therefore, as opposed to existing options, the buyer of an EDS: (1) postpones payment for protection, and (2) purchases not only protection against a sharp drop in the price of equity, but also the right to cease payments in case the barrier is hit. This paper derives the closed-form pricing formula for equity default swap spreads under the Black-Scholes assumptions, and then quantifies the fraction of the EDS spread actually due to the "swap" feature of the contract for plausible parameter values. It is found that the extra spread due to the swap nature of EDSs is economically significant only for high volatility, high trigger levels, and long time-to-maturity. The impact of interest rates on the value of the "swap" feature is almost exclusively due to the postponment of payments.

## 1 Introduction

Equity default swap (EDS) contracts were included in December 2003 for the first time in a rated collateralized debt obligation (CDO). With credit spreads falling steadily, this was viewed as an attempt to boost credit investor returns, since typical EDS spreads are ten times those of credit default swaps (CDSs). An EDS has the same structure as a CDS except the trigger is not a credit event but a drop in the stock price of a reference entity below 30 percent of its value at the inception of the trade. Given that such large drops in stock prices are frequently accompanied by a significant increase in the company's probability of default, it is not surprising that EDSs have been marketed as a hedge against credit risk rather than as an equity instrument. Moreover, extant proposals to price EDSs have been built on the credit risk literature. Medova and Smith (2004), for instance, employ a structural credit model where the company's asset value is the single risk factor driving both the value of equity and the probability of default, whereas Albanese and Chen (2005) develop a credit barrier model where the company's credit quality variable is the driving force for debt and equity value.

Despite their pick-up over credit spreads, the higher transparency with respect to CDSs, and the advantage of knowing the recovery rate in advance (typically, 50 percent of the notional), the innovativeness of EDSs has been doubted by some analysts in the industry (Moore, 2004; Wolcott, 2004). This criticism is understandable since EDSs offer the exact same protection as better known American binary barrier options of the "down-and-in-cash-(at hit)-or-nothing" class. However, EDSs possess two characteristics that differentiate them from existing equity derivatives. First, the trigger

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level is set much lower than in any other equity put or barrier option, which narrows the gap between equity and credit instruments. Second, the "swap" feature implies that the buyer of the EDS makes regular payments instead of paying a single premium upfront, and, also, that the EDS buyer will cease payments if the "equity default" event occurs. It is this "swap" feature that makes EDSs distinct from extant derivatives. Moreover, because this feature is valuable to the protection buyer– due to the postponement of payments and to the option to cease agreed payments after the trigger event has occurred–it enables the seller to increase the spread even further.

The purpose of this paper is two-fold. First, we provide a closed-form solution to the EDS pricing problem under the Black and Scholes (1973) and Merton (1973) assumptions, which is missing in the literature.[1] In order to achieve this goal, we assume that the EDS can be hedged dynamically with a default-free instantaneous bond and a position in the stock, and then use standard risk-neutral valuation techniques to obtain the no-arbitrage EDS spread. Second, we wish to assess quantitatively the innovativeness of EDSs, i.e., we wish to quantify by how much the "swap" feature of EDSs actually increases EDS spreads relative to existing binary barrier options. We do so by comparing the theoretical EDS spread, using our formula, to the theoretical (annualized) premium of an otherwise identical binary barrier option. Additionally, we compare the EDS spread to the spread of a hypothetical binary barrier option whose premium is not paid upfront but on the same payment dates as those specified in the EDS contract. This comparison helps us determine how much the EDS buyer's option to stop paying for protection after the barrier has been hit, is really worth, since that feature is the only difference between the EDS and the hypothetical option.

Numerical comparative statics for realistic parameter values suggests that the "swap" feature is worth less than 20% of the total EDS spread. This fraction, however, becomes large as volatility grows, the distance to equity default shrinks, or the time-to-maturity increases. Interestingly, although high interest rates increase the value of the EDS relative to that of the option, our analysis indicates that this effect is due almost exclusively to the opportunity cost of the premium and, therefore, not attributable to the right to stop making payments after the "equity default" event has occurred. Finally, the frequency of payments appears to have no effect on the value of the "swap" feature.

The rest of the paper is organized as follows: Section 2 derives the pricing formula for EDS spreads under the Black-Scholes assumptions; Section 3 presents comparative statics results; And, finally, Section 4 concludes.

## 2 Pricing equity default swaps in the Black-Scholes world

Throughout this section, we will use the following notation

$S(t)$ : stock price at time $t$;

$\alpha$ : the barrier or trigger level as a fraction of the stock's initial price;

$x$ : fixed recovery rate, as a fraction of the notional ($N$);

$T$ : the EDS maturity date;

$\Phi(\cdot)$ : the cumulative distribution function of the standard normal distribution;

$s_{EDS}$ : The annualized EDS spread as a fraction of the notional. The periodic payment is obtained as $s_{EDS}\Delta N$, where $\Delta$ is the time-length between two consecutive equally-spaced payment dates.

[1] Albanese and Chen (2005) also model the stock price dynamics directly as a pure diffusion process. Because they assume a constant elasticity of variance processs, rather than a geometric Brownian motion, their pricing formula must be implemented numerically. They find that this formula fits the empirically observed EDS to CDS spread ratios more closely than a credit barrier model which incorporates credit jumps and jumps to default. They conclude that jumps do not appear to be priced in the EDS market.

We next derive the theoretical EDS spread in the Black-Scholes world, where the following assumptions hold:

1. markets are perfect (continuous trading is possible and costless);
2. it is possible to invest in a default-free bond maturing instantaneously;
3. the dynamics of the underlying stock price is governed by a geometric Brownian motion under the risk-neutral measure with drift equal to the instantaneous risk-less interest rate, $r$, minus the dividend yield,[2] $q$, and constant diffusion term $\sigma$ :

$$\frac{dS(t)}{S(t)} = (r-q)dt + \sigma dW(t) \tag{1}$$

where $dW(t)$ is a standard Wiener process and $\sigma > 0$.

Under these conditions, it is well known that absence of arbitrage implies that the market value of any future contingent payoff must equal its expected value discounted at the risk-free rate, where the expectation is taken with respect to the risk-neutral measure.

For our purposes, we first need to define the first passage time and its associated density. The first passage time is the first time the stock price hits the barrier and is denoted by $\tau$ :

$$\tau \equiv \inf \{t \mid S(t) \leq B\}$$

Its associated density is defined as:

$$h(t) = \frac{\partial H(t)}{\partial t}$$

where $H(t)$ is the probability of the stock price hitting the barrier before $t$, i.e., $H(t) = \Pr(\tau < t)$. Under the process (1) there is a closed form expression for $h(t)$ (see, for instance, Reiner and Rubinstein, 1991):

$$h(t) = \frac{-\ln(\alpha)}{\sigma t \sqrt{2\pi t}} e^{-\frac{1}{2}\left(\frac{-\ln(\alpha)+\mu t}{\sigma\sqrt{t}}\right)^2} \tag{2}$$

where $\mu \equiv r - q - \frac{\sigma^2}{2}$.

The floating leg in the EDS is the payoff that the protection buyer obtains in case the stock price falls below the barrier before maturity. It is exactly the payoff of a binary barrier option of the "down-and-in-cash-(at hit)-or-nothing" class. Denoting by $\tilde{V}(0,T,xN)$ the market value of the floating leg at the time of initiating the contract, we have:

$$\tilde{V}(0,T,xN) = xN\tilde{V}(0,T,1) = xN \int_0^T e^{-rs} h(s) ds \tag{3}$$

Substituting (2) in (3) gives (Reiner and Rubinstein, 1991):

$$\tilde{V}(0,T,xN) = xN \left[\alpha^{a+b}\Phi(z(T)) + \alpha^{a-b}\Phi(z(T) - 2b\sigma\sqrt{T})\right] \tag{4}$$

where $z(t) \equiv \frac{\ln(\alpha)+b\sigma^2 t}{\sigma\sqrt{t}}$, $a \equiv \frac{\mu}{\sigma^2}$, and $b \equiv \frac{\sqrt{2r\sigma^2+\mu^2}}{\sigma^2}$.

In return for the insurance against the stock price hitting the barrier, the protection buyer must meet a series of periodic payments of $s_{EDS}\Delta N$ on fixed dates ($t_i$, $i = 1, \cdots, n$ with $t_0 = 0$ and $t_n = T$) until the "equity default" event occurs. On that date, if it ever takes place, the protection

[2] See Merton (1973) for an extension of Black and Scholes (1973) to the case of options on dividend paying stock with constant continuously compounded dividend yield.

buyer will also make an accrual payment of the proportional fraction of $s_{EDS}\Delta N$ that corresponds to the length of time since last periodic payment. Such payments constitute the fixed leg in the EDS. The no-arbitrage value of the fixed leg is then obtained as:

$$\begin{aligned}\bar{V}(0,T,s_{EDS}\Delta N) &= \sum_{i=1}^{n} e^{-rt_i} s_{EDS}\Delta N(1-H(t_i)) + \sum_{i=0}^{n-1}\left(\int_{t_i}^{t_{i+1}} e^{-rs}\frac{s-t_i}{\Delta} s_{EDS}\Delta N h(s)ds\right) \\ &= s_{EDS}\Delta N\left[\sum_{i=1}^{n} e^{-rt_i}(1-H(t_i)) + \frac{1}{\Delta}\sum_{i=0}^{n-1}\left(\int_{t_i}^{t_{i+1}} e^{-rs}(s-t_i)h(s)ds\right)\right] \quad (5)\end{aligned}$$

The periodic payment term in (5) requires knowledge of $H(t)$ which is given by the following expression:

$$\begin{aligned}H(t) &= \int_0^t h(s)ds \\ &= \alpha^{2a}\Phi(w(t)) + \Phi\left(w(t) - 2a\sigma\sqrt{t}\right)\end{aligned}$$

with $w(t) \equiv \frac{\ln(\alpha)+\mu t}{\sigma\sqrt{t}}$.

The accrual payment term, on the other hand, requires solving:

$$\int_{t_i}^{t_{i+1}} e^{-rs}(s-t_i)h(s)ds = \int_{t_i}^{t_{i+1}} e^{-rs}sh(s)ds - t_i\int_{t_i}^{t_{i+1}} e^{-rs}h(s)ds$$

for $i = 0, \cdots, n-1$. After some algebra,

$$\begin{aligned}\int_{t_i}^{t_{i+1}} e^{-rs}(s-t_i)h(s)ds &= \frac{-\ln(\alpha)}{b\sigma^2}\left(W(0,t_{i+1}) - W(0,t_i)\right) \\ &\quad -t_i\left(\tilde{V}(0,t_{i+1},1) - \tilde{V}(0,t_i,1)\right) \quad (6)\end{aligned}$$

where:

$$W(0,t) \equiv \alpha^{a+b}\Phi(z(t)) - \alpha^{a-b}\Phi(z(t) - 2b\sigma\sqrt{t})$$

The accrual payment then follows from (6):

$$\begin{aligned}\frac{1}{\Delta}\sum_{i=0}^{n-1}\left(\int_{t_i}^{t_{i+1}} e^{-rs}(s-t_i)h(s)ds\right) &= \frac{-\ln(\alpha)}{\Delta b\sigma^2}\sum_{i=0}^{n-1}\left(W(0,t_{i+1}) - W(0,t_i)\right) \\ &\quad -\frac{1}{\Delta}\sum_{i=0}^{n-1} t_i\left(\tilde{V}(0,t_{i+1},1) - \tilde{V}(0,t_i,1)\right) \\ &= \frac{-\ln(\alpha)}{\Delta b\sigma^2} W(0,T) \\ &\quad -\frac{1}{\Delta}\left((t_{n-1})\tilde{V}(0,T,1) - \Delta\sum_{i=1}^{n-1}\tilde{V}(0,t_i,1)\right)\end{aligned}$$

Finally, for the EDS to have zero value at the beginning of the contract, $s_{EDS}\Delta N$ must be set such that the market value of the floating leg equals the market value of the fixed leg:

$$xN\tilde{V}(0,T,1) = s_{EDS}\Delta N\bar{V}(0,T,1) \tag{7}$$

So,

$$s_{EDS} = \frac{x}{\Delta}\frac{\tilde{V}(0,T,1)}{\bar{V}(0,T,1)}$$

Noting that the stock's initial value enters the formula only through the trigger level as a fraction of the stock's initial price, i.e., $\alpha$, it is evident that EDS spreads are independent of the stock price for fixed $\alpha$. For other diffusions, however, $S(0)$ may affect the stock's local volatility, and hence, the density $h(s)$ of the first hitting time.

The buyer of an otherwise identical binary barrier option would have to pay the premium upfront, which should equal $xN\tilde{V}(0,T,1)$. In order for this premium to be comparable to the EDS spread, it must be expressed as a fraction of the notional value and divided by the length of the contract. Denoting by $s_{OPT}$ this theoretical option spread, we have:

$$s_{OPT} = x\frac{\tilde{V}(0,T,1)}{T}$$

The difference between $s_{EDS}$ and $s_{OPT}$ therefore captures the value of the "swap" feature in EDS contracts, which has two components. The first component is due to the time-value of money: in the option contract, the premium must be paid upfront so the opportunity cost of the premium is lost, whereas in an EDS, payment for protection is split in periodic installments until maturity or until the barrier has been hit. This distinction increases EDS spreads relative to the option premium. The second component corresponds to the second property of the "swap" feature, i.e., the protection buyer's option to stop paying after the barrier has been hit. In order to extract the value of the second component, let us assume the existence of an otherwise identical binary barrier option contract where the buyer splits the premium in periodic payments due on the same dates as in the EDS. The only difference between the EDS and such hypothetical option would therefore be the obligation of the option buyer to meet all remaining payments while the buyer of the EDS only makes the accrual payment.

Denoting by $s_{HOPT}$ the theoretical spread of the hypothetical option, we have a condition equivalent to (7):

$$xN\tilde{V}(0,T,1) = s_{HOPT}\Delta N\frac{1-e^{-rT}}{e^{r\Delta}-1}$$

It thus follows:

$$s_{HOPT} = \frac{x}{\Delta}\tilde{V}(0,T,1)\frac{e^{r\Delta}-1}{1-e^{-rT}}$$

# 3 Comparative statics

For the purpose of quantifying the difference $s_{EDS} - s_{OPT}$, we consider a benchmark case where the constant local volatility term is set at 30 percent, the equity default event takes place when the stock price drops by 70 percent of its initial price, time-to-maturity is five years, the constant riskless interest rate is 3 percent, the EDS buyer is assumed to make semiannual payments, and the continuously compounded dividend yield equals 1 percent. For this benchmark case, we obtain $s_{EDS} = 100.58$ basis points (bp), and $s_{OPT} = 90.16$ bp, so the "swap" feature's contribution to the total EDS spread is 10.36 percent of the total spread. The spread of the hypothetical option with periodic payments, $s_{HOPT}$, equals 97.82, which implies that only 2.74 percent of the EDS spread is attributable to the buyer's option to stop paying after the barrier has been hit, while the rest of the swap's value, 7.62 percent of the EDS spread, is explained by the opportunity cost of the option premium.

The top panels in Figures 1 to 5 plot $s_{EDS}$, $s_{OPT}$, and $s_{HOPT}$ in basis points as functions of one of the model parameters, holding the rest of the variables fixed. The bottom panels show the fraction of the EDS spread explained by the "swap" feature, i.e., $\frac{s_{EDS}-s_{OPT}}{s_{EDS}}$, as well as the fraction of the EDS spread attributable to the option to avoid payments after the default event, i.e, $\frac{s_{EDS}-s_{HOPT}}{s_{EDS}}$.

Figure 1 shows the effect of changes in the constant local volatility parameter. Although the swap's contribution is below 26 percent of the total EDS spread, an increase in $\sigma$, which impacts the likelihood of the stock price hitting the barrier early, not only increases the EDS spread, but also the value of the "swap" feature. For example, when $\sigma$ increases from 30 percent to 40 percent, the difference between the EDS spread and the option "spread" increases from 10 bp to 50 bp, or from 10 to 17 percent of the total EDS spread. The Figure also shows that for volatilities below 40 percent, most of this contribution is attributable to the opportunity cost of the premium, and only for high volatilities does the option to cease periodic payments become relatively valuable, with a maximum contribution of about 35 bp of the EDS spread or less than 20 percent in relative terms.

Very similar effects are found when increasing the trigger level, i.e., decreasing the distance to equity default (Figure 2), since a higher trigger level also contributes to increasing the risk of "equity default". The consequences are especially dramatic for trigger levels above 70 percent of the stock's initial prices (less than 30 percent decline). In that case, the EDS spread is about 1,200 bp, with 500 bp due to the swap. As the trigger level increases, $s_{OPT}$ and $s_{HOPT}$ converge, which indicates that the option to cease payments after default accounts for almost all of the swap's contribution.

Increasing the EDS time-to-maturity (Figure 3) has a smaller impact: doubling the number of years to maturity from the usual 5 to 10, makes the value of the "swap" feature increase from 10 bp to more than 30 bp, or from 10 percent to more than 20 percent of the total EDS spread. In contrast to Figures 1 and 2, however, time-to-maturity has a stronger relative effect on the time-value component of the "swap" feature than on the value of the option to cease payments, with most of the swap's value being due to the first component for all parameter values considered.

Figure 4 shows that increases in the riskless interest rate reduce all spreads. The bottom panel suggests, however, that the relative value of the swap increases sharply as the riskless interest rate grows. This increase is exclusively the consequence of an increase in the opportunity cost of the option premium which makes the EDS relatively more attractive, but hardly affects the value of the option to cease payments (below 5 percent of the EDS spread).

Finally, Figure 5 displays spreads as a function of the number of periodic payments per year. It is clear from the Figure that the frequency of payments does not affect the EDS spread, the relative contribution of the "swap" feature or its composition.

# 4 Conclusions

As opposed to already existing binary barrier options, equity default swaps not only provide the buyer with protection against large declines in the value of equity, but also with a distinct "swap" characteristic. This characteristic, familiar to credit default swap investors, enables the protection buyer to make regular payments rather than a single upfront payment. Moreover, the buyer can cease agreed payments upon the stock price hitting the barrier. In this paper, we have used a closed-form Black-Scholes pricing formula to investigate how valuable this feature can be in terms of its contribution to total EDS spreads, and therefore, how different EDSs are–in quantitative terms–from already existing derivatives.

The main conclusion of the paper is that the "swap" feature has the potential to contribute to less than 20 percent of EDS spreads for plausible parameter values. However, its relative value is higher the higher the volatility of the stock price process, the shorter the distance from the stock's initial price to the trigger level, and the higher the contract's time-to-maturity. Higher interest rates also increase the relative value of the "swap" embedded in the EDS, even though they do so by increasing the opportunity cost of the option and not the value of the option to cease periodic payments. Finally, both EDS spreads and the swap's value are almost insensitive to changes in the frequency of swap payments.

Two interesting questions are left for future work. The first one is whether the conclusions in this paper hold for more realistic diffusions, such as jump-diffusions, constant elasticity of variance, or stochastic volatility processes. The second question is concerned with the empirical validity of theoretical predictions.

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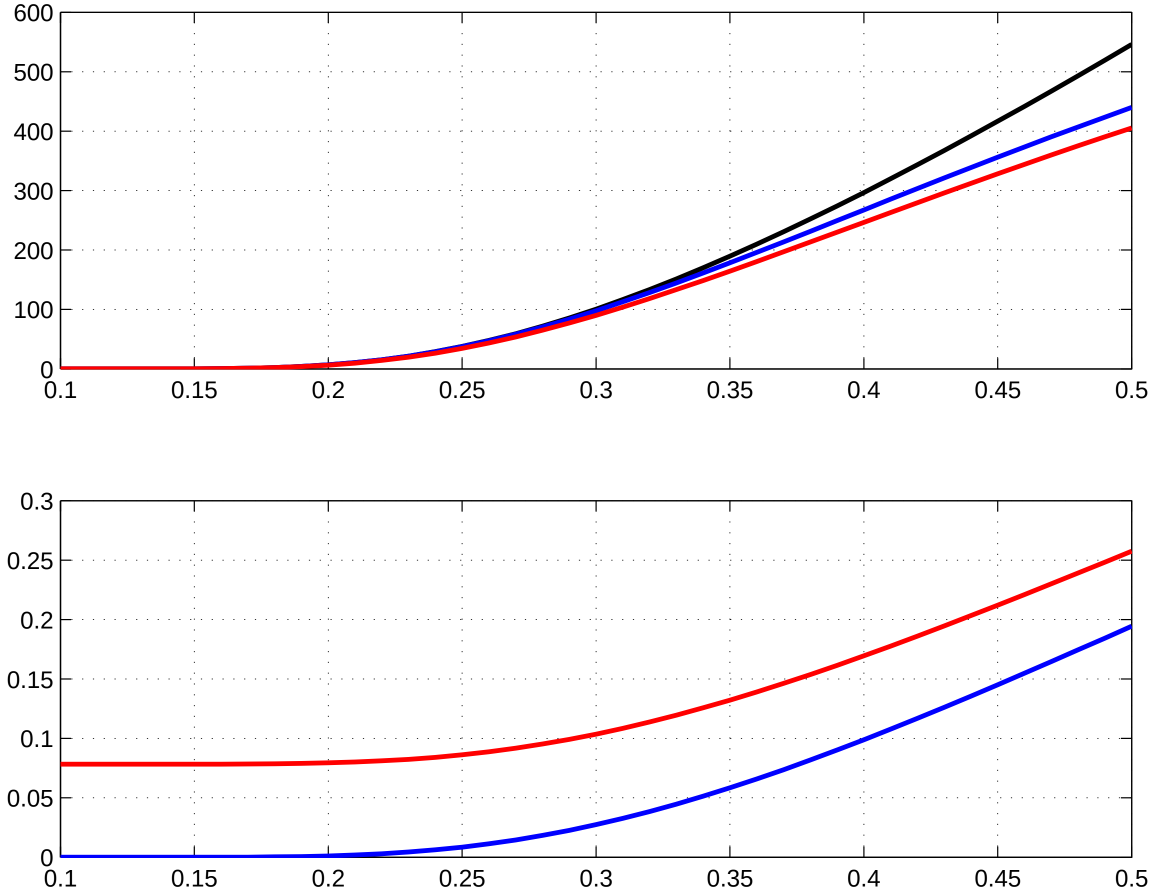


**Figure 1. Effect of changes in volatility.** The top panel shows (in basis points) the EDS theoretical spread (black line), the theoretical spread (annualized premium as a fraction of the notional as a fraction of the notional) of an otherwise identical binary barrier option (red line), and the theoretical spread of a hypothetical binary barrier option with periodic payments instead of an upfront premium, as functions of the constant local volatility parameter. The bottom panel displays the difference between the EDS spread and the option spread as a fraction of the total EDS spread (red line) and the difference between the EDS spread and the hypothetical option spread as a fraction of the total EDS spread (blue line).

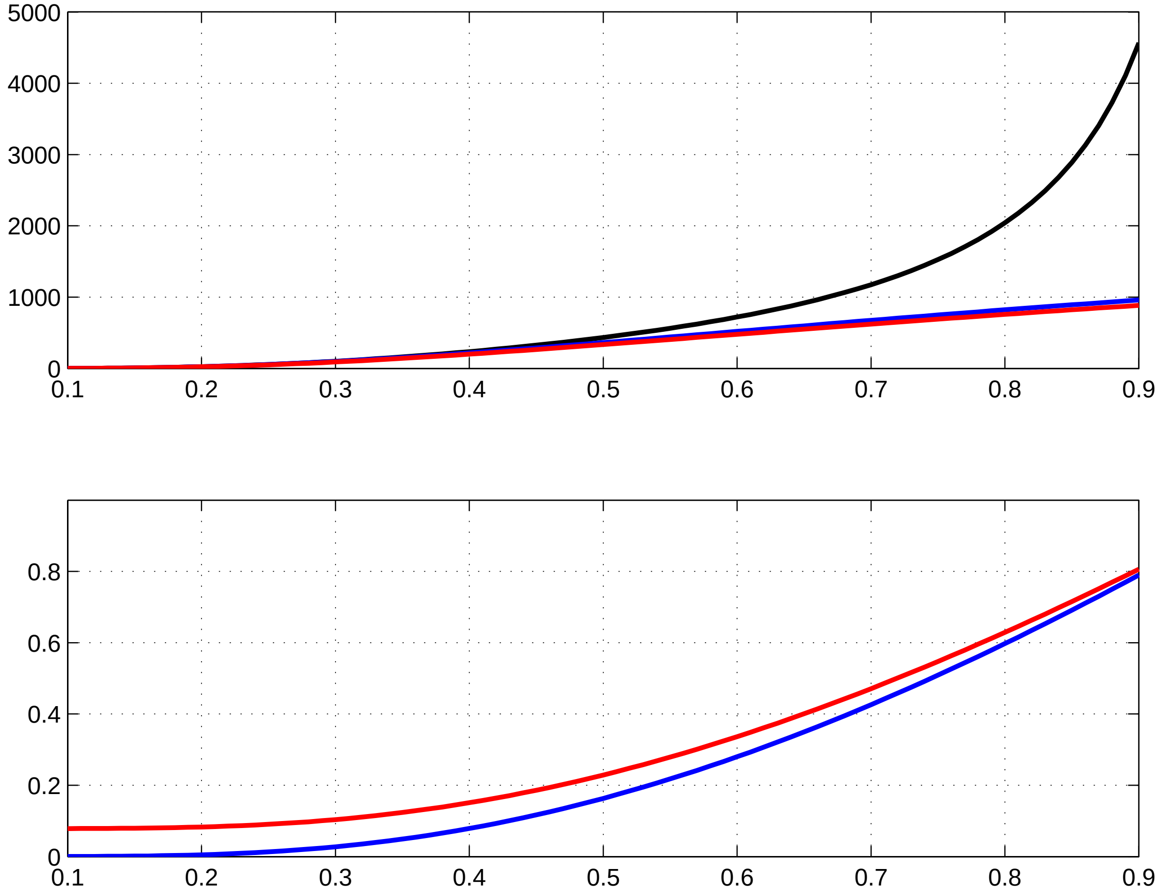


**Figure 2. Effect of changes in the trigger level.** The top panel shows (in basis points) the EDS theoretical spread (black line), the theoretical spread (annualized premium as a fraction of the notional) of an otherwise identical binary barrier option (red line), and the theoretical spread of a hypothetical binary barrier option with periodic payments instead of an upfront premium, as functions of the trigger level as a fraction of the stock's initial price. The bottom panel displays the difference between the EDS spread and the option spread as a fraction of the total EDS spread (red line) and the difference between the EDS spread and the hypothetical option spread as a fraction of the total EDS spread (blue line).

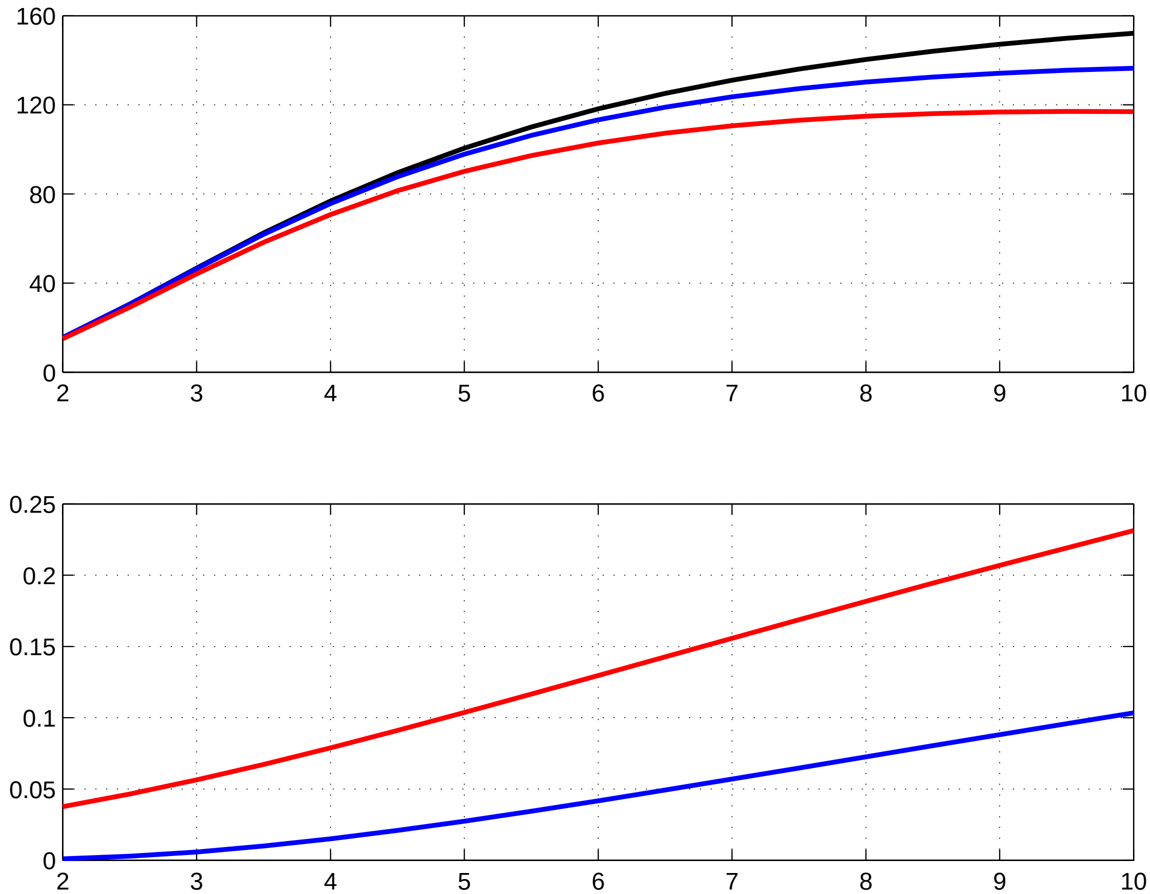


**Figure 3. Effect of changes in term-to-maturity.** The top panel shows (in basis points) the EDS theoretical spread (black line), the theoretical spread (annualized premium as a fraction of the notional) of an otherwise identical binary barrier option (red line), and the theoretical spread of a hypothetical binary barrier option with periodic payments instead of an upfront premium, as functions of the contract's term-to-maturity in years. The bottom panel displays the difference between the EDS spread and the option spread as a fraction of the total EDS spread (red line) and the difference between the EDS spread and the hypothetical option spread as a fraction of the total EDS spread (blue line).

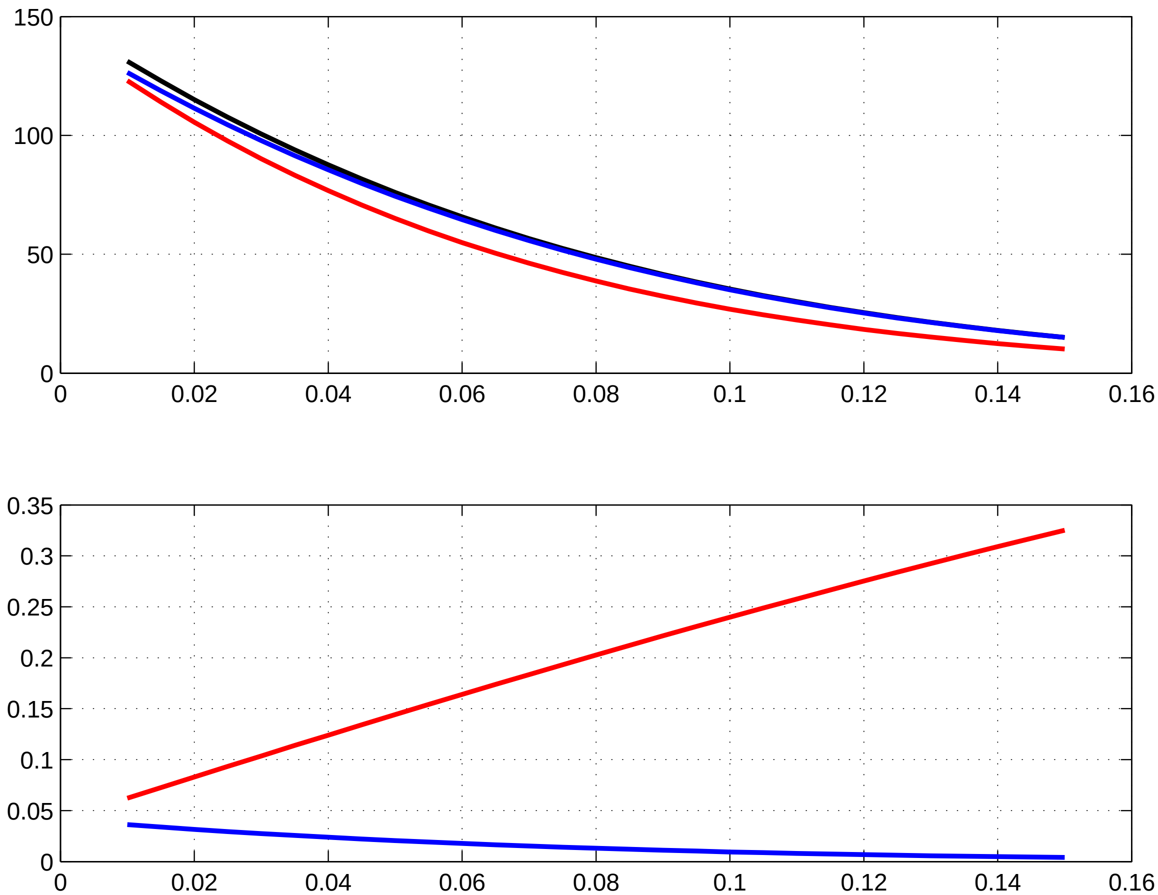


**Figure 4. Effect of changes in the riskless interest rate.** The top panel shows (in basis points) the EDS theoretical spread (black line), the theoretical spread (annualized premium as a fraction of the notional) of an otherwise identical binary barrier option (red line), and the theoretical spread of a hypothetical binary barrier option with periodic payments instead of an upfront premium, as functions of the riskless interest rate. The bottom panel displays the difference between the EDS spread and the option spread as a fraction of the total EDS spread (red line) and the difference between the EDS spread and the hypothetical option spread as a fraction of the total EDS spread (blue line).

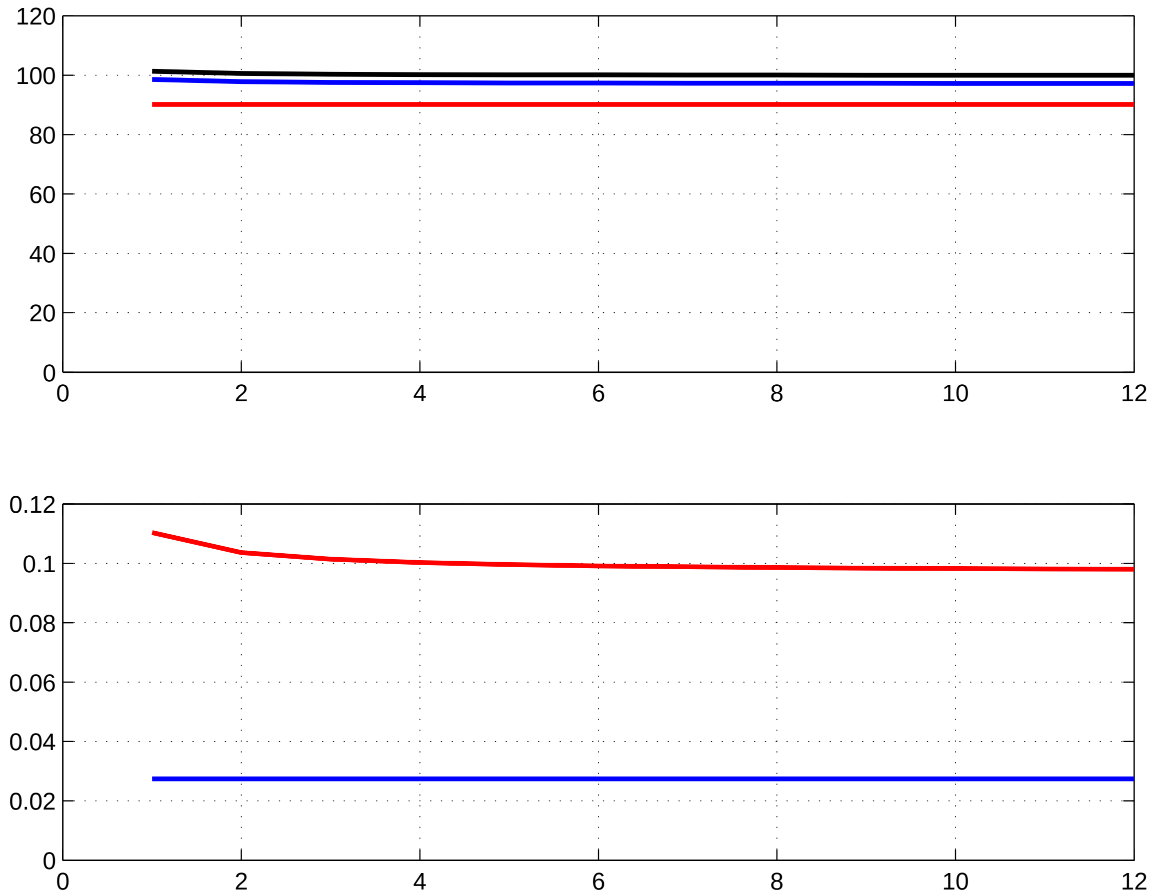


**Figure 5. Effect of changes in the frequency of payments.** The top panel shows (in basis points) the EDS theoretical spread (black line), the theoretical spread (annualized premium as a fraction of the notional) of an otherwise identical binary barrier option (red line), and the theoretical spread of a hypothetical binary barrier option with periodic payments instead of an upfront premium, as functions of the number of payments per year. The bottom panel displays the difference between the EDS spread and the option spread as a fraction of the total EDS spread (red line) and the difference between the EDS spread and the hypothetical option spread as a fraction of the total EDS spread (blue line).